

Assignment 3:

Special Topics related to
Electricity. Simple Harmonic Oscillator

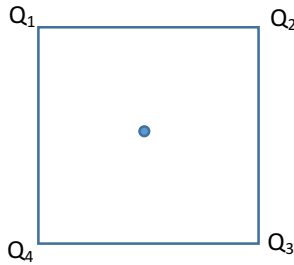
STUDENT #: _____

NAME: _____

Released: Friday Jan. 27

Due: Fri Feb 3 6:00PM

- 1 4 charges of $Q_1=1\text{nC}$, $Q_2=2\text{nC}$, $Q_3=3\text{nC}$, and $Q_4=4\text{nC}$ are placed at the vertices of a square of side 1m. Q_1 is in the top left corner, Q_2 is in the top right corner, Q_3 is in the bottom right corner and the Q_4 in the bottom left corner. Find the magnitude of electric field at the center of the square.
On the Q_4 - Q_2 diagonal the Field E is pointing toward Q_2 and has magnitude of 36N.
On the Q_1 - Q_3 diagonal the Field E is pointing toward Q_1 and has magnitude of 36N.

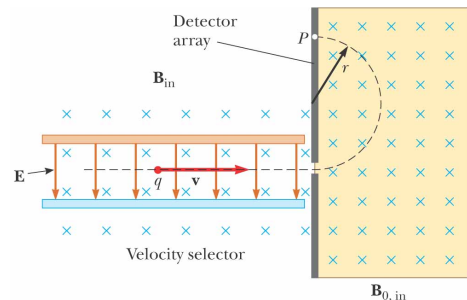


The magnitude of the force is given by the $\sqrt{36^2 + 36^2} = 50.9\text{N}$

- 2 X-Ray lamp accelerates the electrons in the potential difference of 2kV. Find the lowest wavelength of the X rays emitted by this lamp.

$$eV = h \frac{c}{\lambda_{\min}} \Rightarrow \lambda_{\min} = h \frac{c}{eV} = \frac{6.626 \times 10^{-34} \cdot 2.998 \times 10^8}{2000 \times 1.6 \times 10^{-19}} \text{m} = 0.0062 \times 10^{-7} \text{m} = 0.62 \text{nm}$$

- 3 In the standard Mass Spectrometer Setup before entering the magnetic field zone where its trajectory is determined by the Lorentz Force the charged particles pass through the velocity selector. In it, the electric force acts on particles in the direction opposite to the magnetic force. After travelling along the straight line in the velocity selector employing E-field of magnitude of 1000N/C, the carbon ion ($^{12}\text{C}^+$) travels inside the spectrometer along the circular path of radius $r = 1.5\text{m}$. Find the velocity of the ion as it leave the velocity selector.



SOLUTION:

In the Wien's Filter:

$$eE = evB$$

In the Magnetic field zone:

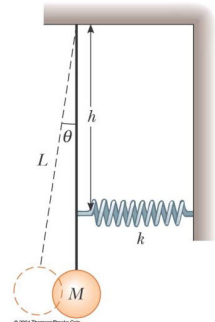
$$evB = \frac{mv^2}{r}$$

Combining the two equations: $evB = \frac{mv^2}{r} \Rightarrow eB = \frac{mv}{r} \Rightarrow \frac{eE}{v} = \frac{mv}{r} \Rightarrow mv^2 = eEr$

$$\Rightarrow v = \sqrt{\frac{eEr}{m}} = \sqrt{0.12 \times 10^{11}} \text{m/s} = 109.5 \text{ km/s}$$

- 4 A pendulum of length L and mass M has a spring of force constant k connected to it at a distance h below its point of suspension. Find the frequency of vibration of the system for small values of the amplitude (small θ). Assume the vertical suspension of length L is rigid, but ignore its mass.

/Use the opposite side of this page to show your solution/



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5 Fill the table for each of the cases of the physical pendulum,

Pendulum type	CM- Pivot distance	Moment of Inertia	Omega	Period
Rod of length L and mass M suspended from a thin light rope at distance r from one end.	$d = L + r$	$I = \frac{1}{12}ML^2 + M(r + \frac{L}{2})^2$	$\sqrt{\frac{Mg(L+r)}{\frac{1}{12}ML^2 + M(r + \frac{L}{2})^2}}$	$2\pi\sqrt{\frac{\frac{1}{12}ML^2 + M(r + \frac{L}{2})^2}{Mg(L+r)}}$
Full sphere of radius R and mass M pivoted at the thin rod piercing it at R/3 from the center.	$d = \frac{R}{3}$	$I = \frac{2}{5}MR^2 + M(\frac{R}{3})^2$	$\sqrt{\frac{Mg\frac{R}{3}}{\frac{24}{45}MR^2}}$	$2\pi\sqrt{\frac{24R}{15g}}$
Disk of radius R and mass m suspended from the point on its edge on the thin long rope of length L.	$d = L + R$	$I = \frac{1}{2}mR^2 + m(L + R)^2$	$\sqrt{\frac{mg(L+R)}{\frac{1}{2}mR^2 + m(L+R)^2}}$	$2\pi\sqrt{\frac{\frac{1}{2}mR^2 + m(L+R)^2}{mg(L+R)}}$
Hoop of radius r and mass m pivoted about the point on its rim.	$d = r$	$I = mr^2 + mr^2 = 2mr^2$	$\sqrt{\frac{mgr}{2mr^2}} = \sqrt{\frac{g}{2r}}$	$2\pi\sqrt{2\frac{r}{g}}$

6

Straight wooden stick of mass M, Length L, uniform cross-section A, and constant density ρ_{wood} with as small mass m attached to its one end, is partially submerged in water (density ρ_{wat}). While in equilibrium stick will floats in vertical position with large part f it submerged. The stick is then pushed down by distance y_{max} from equilibrium and released. (assume no drag forces are acting in this situation),

A) Using first principles demonstrate that such system can be treated as Simple Harmonic Oscillator.

B) Find its period of oscillations.

/Use opposite side of this page to show your solution/

FROM FIRST PRINCIPLES:

(the total mass of the system) x Acceleration = - Buoyant force acting on extra volume of the submerged stick (length y)

$$(M + m) \frac{d^2y}{dt^2} = -A\rho_{\text{wat}}gy \Rightarrow \frac{d^2y}{dt^2} = -\frac{A\rho_{\text{wat}}g}{(M+m)}y \text{ this is SHO Equation}$$

$$T = 2\pi\sqrt{\frac{A\rho_{\text{wat}}g}{(M+m)}}$$

4 solution

We draw a free-body diagram of the pendulum. The force \mathbf{H} exerted by the hinge causes no torque about the axis of rotation.

$$\tau = I\alpha \quad \text{and} \quad \frac{d^2 q}{dt^2} = -\alpha$$

$$\tau = MgL \sin q + kxh \cos q = -I \frac{d^2 q}{dt^2}$$

For small amplitude vibrations, use the approximations: $\sin q \approx q$, $\cos q \approx 1$, and $x \approx s = hq$.

Therefore,

$$\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$$

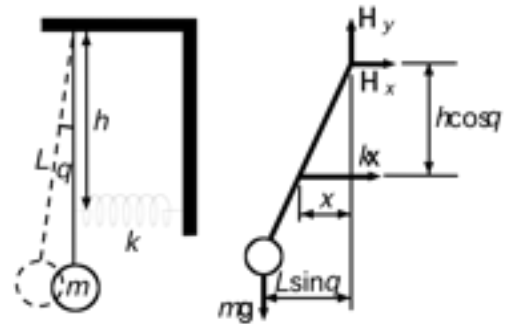


FIG. P15.59

$$\frac{d^2 q}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)q = -\omega^2 q$$

$$f = \frac{1}{2\pi} \sqrt{\frac{MgL + kh^2}{ML^2}}$$